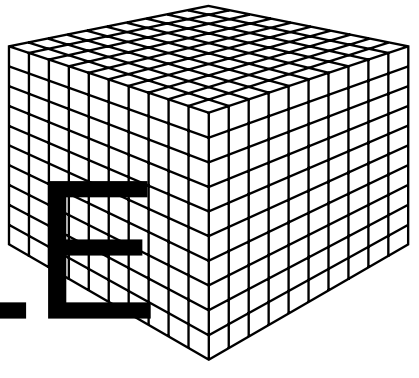
Drilled and painted cubes

A wooden cube of side 10cm is painted red all over and is then cut up into 1000 1cm cubes, as in the figure.



1 How many of these little cubes have

(a) no red face

(b) 1 red face

(c) 2 red faces

(d) 3 red faces?

(e) Find the mean, median and mode of the number of red faces per cube.

SAMPLE

2 If a small cube is picked at random, what is the probability that it has

(a) just one red face,

(b) at least one red face?

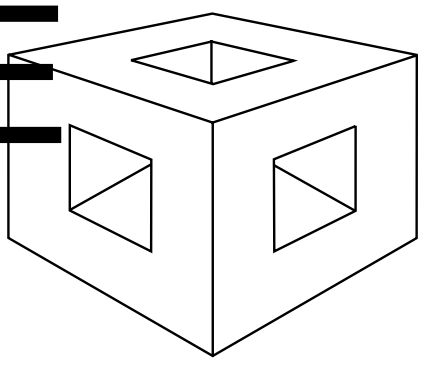
3 If three cubes are taken randomly without replacement, what is the probability that they have just three red faces between them? (Be careful to take account of all possibilities.)

4 Redo no.1(a)–(d) starting with a cube of side n cm cut up into n^3 small cubes, giving answers in the simplest possible form.

(e) What is the mean number of red faces?

(f) For which values of n is 0 the only mode?

5 A cube of side 3 cm has three holes drilled centrally right through, each hole having 1cm square cross-section, as in the figure. Calculate the total surface area and the volume of what remains.



SAMPLE

6 Redo no.5 with

(a) a cube of side 10cm and holes of side 2cm,

(b) a cube of side a cm and holes of side b cm. Simplify and factorise your answers as much as you can. Use the results of nos. 5 and 6(a) to check your formulae.

7 The drilled cube in no.5 is dipped in red paint, allowed to dry and then sawn up into 1cm cubes. How many red faces can a cube have? How many cubes are there of each sort?

8 If the cube in no.6(b) is to be treated similarly, what are the conditions on a and b for this to be possible? Assuming the conditions are fulfilled, find a formula for the mean number of red faces per cube, and show that this gives a mean of $\frac{6}{7}$ for the cube in no.6(a).

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